Chapter 9 Summary

Key Terms
- dilation (9.1)
- center of dilation (9.1)
- scale factor (9.1)
- dilation factor (9.1)
- enlargement (9.1)
- reduction (9.1)
- similar triangles (9.2)
- AA Similarity Theorem (9.3)
- SAS Similarity Theorem (9.3)
- SSS Similarity Theorem (9.3)

Dilating Triangles

Dilations are transformations that produce images that are the same shape as the original image, but not the same size. Each point on the original figure is moved along a straight line and the straight line is drawn from a fixed point known as the center of dilation. The scale factor is the ratio formed when comparing the distance of the image from the center of dilation to the distance of the original figure from the center of dilation.

Example

Enlarge triangle ABC with P as the center of dilation and a scale factor of 2.

First, measure PA. Then, extend the line PA to the point A’ such that \( PA' = 2PA \).

Next, measure PB. Then, extend the line PB to the point B’ such that \( PB' = 2PB \).

Finally, measure PC. Then, extend the line PC to the point C’ such that \( PC' = 2PC \).

Because the scale factor is greater than one, the image \( A'B'C' \) is called an enlargement.

If the scale factor had been less than one, the image would be called a reduction.
9.2 Properties of Similar Triangles

Similar triangles are triangles that have the same shape.

Example

In the figure shown, triangle $DEF$ is similar to triangle $D'E'F'$. This can be expressed using symbols as $\triangle DEF \sim \triangle D'E'F'$.

1. Identify the congruent corresponding angles.
   
   $\angle D \equiv \angle D'$
   
   $\angle E \equiv \angle E'$
   
   $\angle F \equiv \angle F'$

2. Write ratios to identify the proportional sides.

   $\frac{D'E'}{DE} = \frac{E'F'}{EF} = \frac{F'D'}{FD}$

I am going to be an architect and they use these skills all the time. Understanding this already will be a big help when it comes time to go to college.
Using Similar Triangles to Find Unknown Measures

The properties of similar triangles can be used to determine unknown measures of the triangles.

Example

In the figure shown, triangle $ACE$ is similar to triangle $BCD$. Find the length of side $AC$.

Using the proportional relationship between corresponding sides of similar triangles, you know \( \frac{BC}{AC} = \frac{BD}{AE} \).

Substitute the known values for the sides.

\[
\begin{align*}
\frac{9}{AC} &= \frac{6}{12} \\
6AC &= 9(12) \\
6AC &= 108 \\
AC &= 18
\end{align*}
\]

The length of side $AC$ is 18 centimeters.
9.3 AA, SAS, and SSS Similarity Theorems

The Angle-Angle (AA) Similarity Theorem states: “If two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.”

The Side-Angle-Side (SAS) Similarity Theorem states: “If two pairs of corresponding sides of two triangles are proportional and the included angles are congruent, then the triangles are similar.”

The Side-Side-Side (SSS) Similarity Theorem states: “If three pairs of corresponding sides of two triangles are proportional, then the triangles are similar.”

Example

Determine if each pair of triangles are similar by AA, SAS, or SSS.

1. \[ \triangle ABC \sim \triangle FDE \]
   \hspace{1cm} \angle A = \angle F
   \hspace{1cm} \angle B = \angle D
   \hspace{1cm} \triangle ABC \sim \triangle FDE

   The triangles are similar by AA.

2. \[ \triangle ABE \sim \triangle ACD \]
   \hspace{1cm} \frac{AB}{AE} = \frac{AC}{AD}
   \hspace{1cm} \frac{6}{9} = \frac{6}{9}
   \hspace{1cm} \angle A = \angle A

   The triangles are similar by SAS.
The triangles are similar by SSS.

**9.4 Similar Triangles on the Coordinate Plane**

Properties of similar triangles can be used to graph similar triangles, determine the scale factor that was used to create the triangles, and verify that the triangles are similar.

**Example**

Triangle $ABC$ has vertices $A(1,2)$, $B(1,5)$, $C(5,2)$. Graph triangle $ABC$. 

The triangles are similar by SSS.
Triangle DEF is the image that resulted from the dilation of triangle ABC. The coordinates of triangle DEF are D(3,6), E(3, 15), F(15, 6). Graph triangle DEF.

The scale factor used was 3.

Use the SAS Similarity Theorem to verify triangle ABC is similar to triangle DEF.

∠A is congruent to ∠D because they are both right angles and all right angles are congruent.

\[
\frac{AB}{DE} = \frac{AC}{DF}
\]

\[
\frac{3}{9} = \frac{4}{12}
\]

\[
\frac{1}{3} = \frac{1}{3}
\]

The lengths of the sides that include ∠A and ∠D are proportional.

Triangle ABC is similar to triangle DEF by the SAS Similarity Theorem.